2021 QUANTUM TRAJECTORIES FALL SCHOOL: TITLES AND ABSTRACTS

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Tristan Benoist and Clément Pellegrini – **Introduction to quantum trajectories**
Quantum trajectories are Markov processes taking values in density matrices or Hilbert space projective spaces describing the evolution of a quantum system subject to repeated (or continuous) indirect measurements. Although their definition is relatively strightforward, they exhibit some singular behavior. From a physical point of view, they are widely used as modeling tools in quantum optics and quantum stochastic thermodynamics.

In this course, after the definition of the Markov chain associated to such processes and the probability measures defining the laws of the sequences of measurement outcomes, we will present their main properties. We will particularly focus on their long time assymptotic properties (ergodicity, law of large numbers, central limit theorem, large deviations,...). One that is specific to quantum trajectories is the phenomenon of purification. We will particularly see the part it takes in the classification of invariant measures for quantum trajectories. If time remains we will discuss some results in statistics and introduce some current open problems.

Pierre Petit – **Large deviations of the random walk with Weibull-like increments**
The course will focus on the study of the deviations of the random walk. Whereas Cramér's theory provides a beautiful and unified approach in the case of light-tailed increments, many tools cannot be used in the case of heavy-tailed increments. The purpose of the course is to exhibit some general techniques to estimate probabilities of deviations. Here is a brief description of the three lectures.

First, we will review some tools (Markov's inequality, tilting, convexity, subadditivity, contraction principle) that can be used to prove Cramér's theorem for a light-tailed distribution and mention some generalizations (Gärtner-Ellis theorem, Bryc's theorem, asymptotically decoupled fields).

Secondly, we will consider the case of a Weibull-like distribution, the tail of which is asymptotically $\exp(-x^\alpha)$ for some $\alpha \in (0, 1)$. We will see that some techniques are still applicable here. Contrary to the case of light-tailed distributions, sufficiently large deviations are driven by a single large increment, a fact which is common to many heavy-tailed (notably subexponential) distributions.

Thirdly, we will discuss the general framework of a simple random walk S_n conditioned on the fact that another random walk T_n is equal to some value m_n. Many models enter this general framework, including the occupancy problem, branching processes, random forests, Bose-Einstein statistics, and hashing. Although Gärtner-Ellis theorem allows to get a large deviation principle in most cases, the cases of hashing involves Weibull-like distributions and requires a specific treatment that we will expose as an application.

Laurent Bruneau – **Out of equilibrium repeated interaction quantum systems, the iid case.**

Repeated interaction systems consist of a quantum system S interacting with a sequence of probes, one after the other. The physical paradigm of such systems is the one-atom maser where S describes the quantized electromagnetic field in a cavity and the probes form a beam of atoms which are shot in such a way that no more than one atom is present in the cavity at each time. These systemes can be seen as a discrete time version of quantum trajectories or as a particular class of open quantum systems, the sequence of probes representing here the environment. We will consider the open system point of view and will focus on a non-equilibrium situation: the various probes are not always the same but chosen from a finite set of possible ones (one should think of probes all initially in a Gibbs state but with possibly different temperatures). We will consider the situation where the sequence of probes is chosen randomly at each step and in an iid way. We are then interested in the large time behaviour of the system: relaxation to a steady state and questions related to entropy production and energy fluxes (linear response theory and entropic fluctuations).

Ángela Capel Cuevas – **Logarithmic Sobolev Inequalities for Quantum Many-Body Systems**

The mixing time of Markovian dissipative evolutions of open quantum many-body systems can be bounded using optimal constants of certain quantum functional inequalities, such as the logarithmic Sobolev constant. For classical spin systems, the positivity of such constants follows from a mixing condition on the Gibbs measure, via quasi-factorization results for the entropy.

Inspired by the classical case, we present a strategy to derive the positivity of the logarithmic Sobolev constant associated to the dynamics of certain quantum systems from some clustering conditions on the Gibbs state of a local, commuting Hamiltonian. Subsequently, we apply this strategy to address this problem for several dynamics.

Franco Fagnola – **Gaussian Quantum Markov Semigroups on Fock Space: Irreducibility and Normal Invariant States**

We consider the most general Gaussian quantum Markov semigroup the algebra of all bounded operator on a Fock space, discuss its construction from the generalized GKLS representation of the generator. We illustrate the known explicit formula on Weyl operators, irreducibility and its equivalence to a Hormander type condition on commutators and establish necessary and sufficient conditions for existence and uniqueness of normal invariant states. We will see that many properties can be deduced from those of matrices on the one-particle space of the Fock space.

Federico Girotti – **Large deviations, central limit and dynamical phase transitions in the atom maser**

The theory of quantum jump trajectories provides the framework for understanding dynamical phase transitions in open systems. A candidate for such transitions is the atom maser; although previous numerical results suggested that the "free energy" may not be a smooth function, in our talk we show that the atom detection counts satisfy a large deviations principle, and therefore we deal with a phase cross-over rather than a genuine phase transition. As a corollary, we obtain the Central Limit Theorem for the counting process. The proof relies on the analysis of a certain deformed generator whose spectral bound is the limiting cumulant generating function. The latter is shown to be smooth, so that a large deviations principle holds by the Gärtner-Ellis Theorem. One of the main ingredients is the

Krein-Rutman theory which extends the Perron-Frobenius theorem to a general class of positive compact semigroups. The talk is based on a joint work with R. Carbone, M. Guţă and M. van Horssen.

Carlos Gonzalez-Guillen – **Spectral gap of random quantum channels**

Quantum channels are the most general linear transformations quantum systems can undergo. As such, the study of their properties is of central importance in Quantum Information Theory. In the last two decades, there has been an increasing interest in the study of generic quantum channels, that is random quantum channels having natural probability distributions.

In this talk, we will study the singular value and spectral gap of random quantum channels induced by random isometries. In particular, we will concentrate in those with a fixed number of Kraus operators, but very large input and output dimensions. We will show a lower bound on the difference between the first and second singular values of such quantum channels, that is tight in the scaling of the number of Kraus operators. Besides the fundamental importance of answering such questions, we will motivate our result with two applications. We show that these random quantum channels are quantum expanders. As a second application of our results, we prove that ground states of infinite 1D spin chains, which are well-approximated by matrix product states, fulfill a principle of maximum entropy.

Based on work with M. Junge and I. Nechita arxiv:1811.08847

Géraldine Haack – **Dynamics of open quantum systems – an introduction to quantum thermal machines**

In this talk, I will first introduce open quantum systems and a general form for their evolution in time. For Markovian environments weakly coupled to quantum systems, their evolution equations take a Lindblad form that can be derived following a variety of approaches that I will review; from microscopic models for the baths to a Hamiltonian perturbative approach, through the theory of quantum channels or the use of stochastic models. Then, I will study out-of-equilibrium systems, made of a chain of quantum systems, coupled at each end to a Markovian environment. Their steady-state and out-of-equilibirum currents (particle, charge, heat) will be discussed based on Lindblad evolution equations. Interestingly, these open quantum systems constitute paradigmatic models for quantum thermal machines, able to produce work (heat engine), to cool (refrigerator) or to create quantum correlations (entanglement engine). I will use these concrete examples to illustrate the steady-state properties of open quantum systems.

References:

- G. Haack and A. Joye, *Perturbation Analysis of Quantum Reset Models*, J. Stat. Phys. 183 (2021).
- J. Bohr Brask, G. Haack, N. Brunner and M. Huber, *Autonomous quantum thermal machine for generating steady-state entanglement*, New J. Phys. 17 (2015).
- S. Khandelwal, N. Palazzo, N. Brunner and G. Haack, Critical heat current for operating an entanglement engine, New J. Phys. 22 (2020).

Sherya Mehta – **Hypocoercive dissipative dynamics in noncommutative spaces**

Since the noncommutative framework is necessary to understand the events at a quantum scale, we want to develop the noncommutative analogue for the classical Hörmander theory. We will also present the noncommutative setup for the analogue of Hypocoercive functional inequalities introduced by Villani.

Annalisa Panati – **Entropic Fluctuations in quantum two-time Measurement Framework**

Non-equilibrium statistical mechanics has seen some impressive developments in the last three decades, thank to the pioneering works of Evans, Cohen, Morris and Searles on the violation of the second law, soon followed by the ground-breaking formulation of the Fluctuation Theorem by Gallavotti and Cohen for entropy fluctuation in the early nineties. Their work was by vast literature, both theoretical and experimental.

The extension of these results to the quantum setting has turned out to be surprisingly challenging and it is still an undergoing effort. Kurchan's seminal work (2000) showed the measurement role has to be taken in account, leading to the introduction of the so called two-time measurement statistics (also known as full counting statistics). However introducing this frameworks leads to surprising phenomena with no classical counterpart. In this talk, I will present some work in progress, where we attempt to introduce a quantum equivalent of Gallavotti-Cohen (steady) entropic functional and compare it with the Evans-Searls (transient) entropic functional. We show that, due to the invasive measurement role, the situation differs considerably to its classical counterpart.

Joint work with T. Benoist, L. Bruneau, V. Jakšic, C.A.Pillet.

Claude-Alain Pillet – **Markovian Repeated Interactions Systems (joint work with J.-F. Bougron and A. Joye)**

We study a class of dynamical semigroups $(L_n)_{n\in\mathbb{N}}$ that emerge, by a Feynman–Kac type formalism, from a random quantum dynamical system $\mathcal{L}_{omega_n}\cdots\mathcal{L}_{\omega_1}\rho_{\omega_0}$ driven by a Markov chain $(\omega_n)_{n\in\mathbb{N}}$. We show that the almost sure large time behavior of the system can be extracted from the large n asymptotics of the semigroup, which is in turn directly related to the spectral properties of the generator L. As a physical application, we consider the case where the $\mathcal{L}_\omega$'s are the reduced dynamical maps describing the repeated interactions of a system S with thermal probes C_ω . We study the full statistics of the entropy in this system and derive a fluctuation theorem for the heat exchanges and the associated linear response formulas.

Lev-Arcady Sellem – **Model order reduction methods for Lindblad type equations**

We study the *cat qubit* proposal for the physical implementation of a quantum computer, where each qubit is encoded into the state of a quantum harmonic oscillator.

Assessing its suitability – e.g. the provided protection of the encoded logical information from external perturbations, the experimental parameter range to achieve this protection, etc. – relies on our ability to simulate this system, modeled by Lindblad type equations on the Hilbert space $H = L^2(\mathbf{R}, \mathbf{C})$, in practically relevant settings. The main numerical challenges are the infinite dimension of $\mathcal{H}$ and the preservation of the geometric properties defining density operators.

The usual strategy for the simulation of quantum harmonic oscillators is a Galerkin projection onto the space spanned by a finite truncation of the so-called *Fock basis*. This method however quickly leads to untractable computations when applied to the simulation of cat qubits, due to the inability of Fock states to accurately represent typical solutions.

In this talk, we present a systematic strategy to extract an efficient simulation basis from a few (costly) reference simulations in the Fock basis, by a systematic model order reduction strategy adapted to Lindblad type equations on density operators. We compare our strategy to a recently introduced *ad-hoc* strategy taylored specifically to the simulation of cat qubits, and provide numerical evidence that our strategy yields numerical results with similar accuracy and marginal cost in the online stage. This paves the way to an efficient and systematic numerical approach for more complex settings that are being addressed in ongoing works.

This is joint work with Claude Le Bris and Pierre Rouchon.

Andreas Winter – **Equilibration of quantum systems in a pure state**

We will review a certain approach to the foundations of statistical mechanics, in which subjective randomness, ensemble-averaging or time-averaging are not required. Instead, the complete physical system (i.e. the subsystem of interest together with a sufficiently large environment) is in a pure quantum state subject to a global constraint, and thermalisation results from entanglement between system and environment.

In the "kinematic" setting of statistical mechanics, we formulate and prove a "General Canonical Principle", which states that the system will be thermalised for almost all pure states of the universe, and provide rigorous quantitative bounds using Levy's Lemma.

In the second part, we go on to consider a full dynamical model of equilibration in a setting of closed system Hamiltonian dynamics. We find conditions under which initial states equilibrate, and under which the equilibrium state has the character of a canonical state.

[Based mostly on work with S Popescu and T Short, arXiv:quant-ph/0511225; and with N Linden, S Popescu and T Short, arXiv:0812.2385]

Haonan Zhang – **Curvature-dimension conditions for symmetric quantum Markov semigroups**

The curvature-dimension condition consists of the lower Ricci curvature bound and upper dimension bound of the Riemannian manifold, which has a number of geometric consequences and is very helpful in proving many functional inequalities. The Bakry–Émery theory and Lott–Sturm–Villani theory allow to extend this notion beyond the Riemannian manifold setting and have seen great progress in the past decades. In this talk, I will first review several notions around lower Ricci curvature bounds in the noncommutative setting and present our work on complete gradient estimates. Then I will speak about two noncommutative versions of curvature-dimension conditions for symmetric quantum Markov semigroups over matrix algebras. Under suitable such curvature-dimension conditions, we prove a family of dimension-dependent functional inequalities, a version of the Bonnet–Myers theorem, and concavity of entropy power in the noncommutative setting. I will also give some examples satisfying certain curvature-dimension conditions, including Schur multipliers over matrix algebras, Herz-Schur multipliers over group algebras, and depolarizing semigroups. This is based on joint work with Melchior Wirth (IST Austria).